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GRAPHICAL CONSTRUCTIONS FOR A FUNCTION OF A FUNCTION
AND FOR A FUNCTION GIVEN BY A PAIR OF PARAMETRIC
EQUATIONS.\textsuperscript{1}

By W. H. ROEVER, Washington University.

It is easy to see that if \(y = \phi(u)\) and \(u = \psi(x)\) are the equations of the ortho-
graphic projections of a space curve on the planes \(y-O-u\) and \(u-O-x\) determined
by the system of rectangular cartesian axes \(O-x, u, y\), then \(y = \phi[\psi(x)]\) is the
equation of the orthographic projection of this space curve on the plane \(x-O-y\)
(see Fig. 1).

\[ y = \phi[\psi(x)] \]

Let us now suppose the planes \(\pi_1 \equiv xOu\) and \(\pi_3 \equiv uOy\) to be revolved
around the axes of \(x\) and \(y\) respectively until they coincide with the plane \(\pi_2 \equiv yOx\),
and in such a way that the revolved positions of the positive \(u\)-axis coincide with
the negative portions of the unrevolved axes of \(x\) and \(y\) (see Fig. 2). If a general
point of space be denoted by \(P\), let its orthographic projections on \(\pi_1, \pi_2,\) and \(\pi_3\)
be denoted by \(P', P''\) and \(P'''\) respectively. During the revolutions just made
the point \(P''\) remains unchanged, but the points \(P'\) and \(P'''\) assume the new posi-
tions \(P'\) and \(P'''\) respectively (see Fig. 2). The plane \(\pi_2\), which now also con-
tains the revolved positions of the planes \(\pi_1\) and \(\pi_3\), is the drawing plane of Monge-
anean Descriptive Geometry and the unrevolved axes of \(x\) and \(y\) are called the first
and second ground lines respectively (see Fig. 3). The internal bisector of the
right angle formed by the positive portions of these ground lines is called the line \(b\). From the method of obtaining the points \(P', P'', P'''\) of the drawing
plane it follows that:

1. \(P'\) and \(P''\) lie on the same perpendicular to the first ground line,

\textsuperscript{1}Presented to the American Mathematical Society, November 27, 1915.
After this paper was written my attention was called to a construction by Professor E. H.
Moore which is practically identical with the one here given. Professor Moore's construction is
contained under Linkage B in his article "Cross-section paper as a mathematical instrument"
published in The School Review, May, 1906. Dr. A. J. Kempner has also discovered a construction
which includes the one here given. His paper will be published in a later number of this Monthly.
(2) $P''$ and $P'''$ lie on the same perpendicular to the second ground line,
(3) lines through $P'$ and $P''$ parallel respectively to the first and second ground lines, meet in a point $P^o$ of the line $b$ (see Fig. 3). It is important to observe that this is true regardless of the position of $P$ in space; that is, $P$ need not necessarily lie in the first octant as shown in Fig. 2. Following general usage we shall call the points $P'$, $P''$, $P'''$ the projections in the drawing plane of the point $P$ of space. We may then say that no matter what the position of a point in space may be, its projections in the drawing plane are three vertices of a rectangle whose sides are parallel to the ground lines, and whose fourth vertex lies on the line $b$.

By combining the facts stated in the two preceding paragraphs we are led to the Construction (Fig. 3). Assume a pair of perpendicular lines (one horizontal and the other vertical) intersecting in a point $O$. Let these represent respectively the axes of $x$ and $y$, $x$ being positive to the right of $O$ and $y$ positive above $O$. Let each of these lines also represent the axis of $u$, in which however the positive and negative directions are the reverse of those in $x$ and $y$. Then draw through $O$ the line $b$ bisecting internally the angle formed by the positive portions of the axes of $x$ and $y$. Now plot with respect to the axes of $x$ and $u$ the graph of $u = \psi(x)$ and with respect to $u$ and $y$ the graph of $y = \phi(u)$. All this being done, we find the required graph of $y = \phi[\psi(x)]$ by drawing through each point $P^o$ of the line $b$ a horizontal line cutting $u = \psi(x)$ in $P'$ and a vertical line cutting $y = \phi(u)$ in $P'''$, and then through $P'$ a vertical line and through $P'''$ a horizontal line. The last two lines intersect in a point $P''$ of the required graph.

In Fig. 4, this construction is used to find the graph of $y = e^{1/x}$ from the graphs of $y = e^u$ and $u = 1/x$. 
This method also enables one to construct a curve which is given by a pair of parametric equations

\[ x = f(u), \quad y = \phi(u). \]

For the problem of eliminating \( u \) between these equations is the same as that of finding the function \( y = \phi(u) \) of the function \( u = \psi(x) \), where \( u = \psi(x) \) is obtained by solving \( x = f(u) \) for \( u \) in terms of \( x \). In Fig. 5, the method is used to construct the curve of which the parametric equations are

\[ x = u^2, \quad y = u^3. \]

It is interesting to note that for \( u = 0 \) both of the functions \( f(u) = u^2, \phi(u) = u^3 \) have vanishing derivatives and hence the curve \( y^2 = x^3 \) has a singularity—in this case a cusp—at the origin. The curves in Figs. 4 and 5 have been accurately...
constructed. However, the construction under consideration often enables one to determine the salient features of the graph of \( y = \phi[\psi(x)] \) from a mere rough plotting of the graphs of \( y = \phi(u) \) and \( u = \psi(x) \).

**A NOTE ON THE SUM OF THE REMAINDERS OF A SERIES.**

By GLENN JAMES, Purdue University.

Suppose the series

\[
a_0 + a_1 + a_2 + \cdots + a_n + \cdots
\]

has the sum \( S \), and consider the series formed from its remainders, which is

\[
S + [S - a_0] + [S - (a_0 + a_1)] + \cdots + [S - (a_0 + a_1 + \cdots + a_{n-1})] + \cdots
\]

or

\[
R_0 + R_1 + R_2 + \cdots + R_n + \cdots.
\]

The assumption that (1) converges is equivalent to the assumption that

\[
\lim_{n \to \infty} R_n = 0.
\]

Moreover, any convergent or divergent series

\[
H_0 + H_1 + H_2 + \cdots + H_n + \cdots,
\]

\[1\] It might be remarked that the addition of the 45° line \( b \) on ordinary rectangular plotting paper would make of that paper a splendid medium for carrying out the above construction.